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5/ Relativistic 4-vectors. Real Clifford 4-vectors can be made to represent the classical and relativistic kinematic vectors for example path increment $d\mathbb{1} = dt + dx_1 e + dx_2 f + dx_3 g$, energy+momentum 4-vector, electromagnetic potential 4-vector or charge-density + current density 4-vector etc. Complex Clifford vectors represent wave functions. The Clifford product of two wave-function vectors is interpreted as sequential superposition of two quantum events, representing evolution of quantum amplitude of physical processes taking place sequentially. Vector addition of two wave-function vectors represents the combined quantum amplitude of two probable quantum processes taking place simultaneously along different quantum trajectories each, in its parallel multiverse, as per Feynman's diagram model.

Reference: "Standard model physics from an algebra?" by Cohl Furey, PhD thesis, Waterloo, Ontario, Canada, 2015 <https://arxiv.org/pdf/1611.09182.pdf...> Note: Dr. Furey employs Octonions $Cl(6) \times \mathbb{C}$, equivalent to $Cl(7,0)(\mathbb{R})$ based on the notation I am using. $Cl(3,0)(\mathbb{R})$ is isomorphic to Complex Quaternions

6/ Exponential function. Clifford vectors can be exponentiated. Exponent function is defined as the infinite sum

$$\exp(v) := 1 + v + \frac{vv}{2} + \frac{vvv}{6} + \dots + \frac{v^n}{n!} + \dots$$

Important class of arguments are vectors that are linear combinations of bases $\{e, f, g\}$ only, with all Real or all Imaginary coefficients (without the unity scalar term). Such vectors are called transformation 'generator' vectors for the general transformation formula of the relativistic 4-vectors from one coordinate system to another: $a \rightarrow a'$:

$$a' = \exp(v) a \exp(v^T)$$

Note: Operator $()^T$ is called Hermitian Conjugate, defined as an involution function $Cl(3,0)(\mathbb{R}) \rightarrow Cl(3,0)(\mathbb{R})$ that reverses the order of multiplication of two Clifford vectors (that is, for all a, b : $(ab)^T = b^T a^T$) but does not alter the signs of $\{e, f, g\}$. Example: $e^T = e$, $1^T = 1$, $i^T = -i$, $i^T = (ef)^T = -fe = -i$.

7/ Rotation and Lorentz transform of the 4-vectors.

General transform. The following formula

$$a' = \exp(v) a \exp(v^T)$$

produces 3D rotation transform and relativistic (Lorentz) transform of 4-vectors of type $a, b = \{1, e, f, g\} \times \mathbb{R}$ with real coefficients only! [] Minkowski's/Einstein's 4-vectors, for example $a = (t, x, y, z)$

where t, x, y, z are all Real coordinate coefficients, can be represented in Clifford notation as $a = t + xe + yf + zg$. []

3D Rotation uses $v = \{e, f, g\} \times \text{Imaginary}$, resulting in the transformation formula:

$$a' = \exp(v) a \exp(-v).$$

Notice that Clifford modulus of a defined as $|a|^2 = a a^\sim$ is an invariant of the Lorentz and rotation transforms [] :

$$|a|^2 = t^2 - x^2 - y^2 - z^2$$

Note: Operator $()^\sim$ is called Clifford Conjugation or Quaternion Conjugation defined as an involution that flips the signs of bases $\{e, f, g\}$ and reverses the multiplication order. That is, for all a, b : $(ab)^\sim = b^\sim a^\sim$ and it toggles the signs of $\{e, f, g\}$. Example: $e^\sim = -e$, $1^\sim = 1$, $i^\sim = i$, $i^\sim = (ef)^\sim = -fe = -i$.

Example of rotation

Let us use the following transform generator vector $v = i\theta g$: $a' = \exp(i\theta g) a \exp(-i\theta g)$, $a = t + xe + yf + zg$. We need to re-group the terms separating those that are commutative with g , and those that are anti-commutative with g : []

$$a' = \exp(i\theta g)(t + zg)\exp(-i\theta g) + \exp(i\theta g)(xe + yf)\exp(-i\theta g) =$$

$$(t + zg) + \exp(2i\theta g)(xe + yf) =$$

$$(x \cos 2\theta + y \sin 2\theta)e +$$

$$(-x \sin 2\theta + y \cos 2\theta)f + t + zg$$

The first two terms at e and f are identical to a rotation matrix by angle 2θ along the z (g) axis. The scalar t and z components are of course rotationally invariants, in this case.