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Clifford Algebra Twitter Course 27-Feb-2020 - 22-March-2020

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 #cliffordalgebra - A short summary of Clifford Algebra-based methodology as a unified alternative to tensor calculus in classical, relativistic, quantum physics and electromagnetics

1/ Clifford Algebra. Cl3,0(ℝ) can be constructed of a triple base (referred to as 'primary') of anti-commuting operators e,f,g (corresponding to y,z,x in this order). A 'product' of two bases (operators) is interpreted as a superposition of operators. A superposition of the same operator (with itself) gives identity I, that is: ee=ff= gg=I. Superposition of different operators produces 'bivector' bases (also referred to as 'secondary' base) i,j,k that is ef=-fe=i, fg=-gf=j, ge=-eg=k. Note that any and all linear operators in Cl3,0(ℝ) need to be defined only on the three primary bases {e,f,g}, and can be extended on the entire Cl3,0(ℝ) domain using the axioms of the algebra.

2/ Trivector. Trivector is a triple product of base vectors (operators) efg=i . It is commutative with all other base operators (base vectors) and has the same property as the imaginary unit of the Complex number field, that is ii = -I. Full vector base consists of {I,e,f,g,i,j,k,i} . A general Clifford vector can be constructed the same way as in classical Cartesian (or more generally in Hilbert space) vector algebra by linear combination of its base vectors with real coefficients (though it is important to keep in mind that unlike in the former, Clifford base vectors are not generally orthogonal).

3/ Scalar and Imaginary Scalar . Identity and trivector bases {I,i} have the same algebraic properties as the real number 1 (we will write 1 instead of I) and imaginary number i . Thus the sub-algebra {1,i} is isomorphic to Complex number field ℂ allowing self-complexification of Cl3,0(ℝ) = {1,e,f,g,i,j,k,i}xℝ into an equivalent complex version of this algebra {1,e,f,g}x{1,h} or {1,e,f,g}xℂ. [Note]

4/ Full Clifford 8-vectors . Full Clifford vectors represent physical objects called Spinors. Wave-function of a fermion particle is a spinor. A Complex 4-vector **a**= a_0+a_1e+a_2f+a_3g (where a_0,a_1,a_2,a_3 are arbitrary Complex coefficients) . a represents a Dirac spinor. If coefficients a_0,a_1,a_2,a_3 are Real then **a** represents Einstein/Minkowski 4-vector. If a_0,a_1,a_2,a_3 have a common complex scale factor of the type exp(i□□) which can be factored out of each a_i = exp(i□□)b_i where b_i is real, i=0..3, then □□ can be interpreted as the quantum phase of boson particle **a**.

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