2. Expanded

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5/ Relativistic 4-vectors. Real Clifford 4-vectors can be made to represent the classical and relativistic kinematic vectors for example path increment d□□ = $dt+dx_1e+dx_2f+dx_3g$, energy+momentum 4-vector, electromagnetic potential 4-vector or charge-density + current density 4-vector etc. Complex Clifford vectors represent wave functions. The Clifford product of two wave-function vectors is interpreted as sequential superposition of two quantum events, representing evolution of quantum amplitude of physical processes taking place sequentially. Vector addition of two wave-function vectors represents the combined quantum amplitude of two probable quantum processes taking place simultaneously along different quantum trajectories each, in its parallel multiverse, as per Feynman's diagram model.

Reference: "Standard model physics from an algebra?" by Cohl Furey, PhD thesis, Waterloo, Ontario, Canada, 2015 [https://arxiv.org/pdf/1611.09182.pdf...](https://arxiv.org/pdf/1611.09182.pdf) Note: Dr. Furey employs Octonions Cl(6)xC, equivalent to Cl7,0(ℝ) based on the notation I am using. Cl3,0(ℝ) is isomorphic to Complex Quaternions

6/ Exponential function. Clifford vectors can be exponentiated. Exponent function is defined as the infinite sum

exp(v) := 1+v+vv/2+vvv/6+...+vn/n!+...

Important class of arguments are vectors that are linear combinations of bases {e,f,g} only, with all Real or all Imaginary coefficients (without the unity scalar term). Such vectors are called transformation 'generator' vectors for the general transformation formula of the relativistic 4-vectors from one coordinate system to another: a → a' :

a' = exp(v) a exp(vT)

Note: Operator $( )^T$ is called Hermitian Conjugate, defined as an involution function Cl3,0(ℝ) → Cl3,0(ℝ)   that reverses the order of multiplication of two Clifford vectors (that is, for all a,b : (ab)T = bTaT) but does not alter the signs of {e,f,g}. Example: $e^T=e$, $1^T=1$, $i^T=-i$, $i^T=(ef)^T = -fe = -i$.

7/ Rotation and Lorentz transform of the 4-vectors.

General transform. The following formula

$a'=\exp(v)\ a\ \exp(v^T)$

produces 3D rotation transform and relativistic (Lorentz) transform of 4-vectors of type a,b = {1,e,f,g}xℝ with real coefficients only![ ] Minkowski's/Einstein's 4-vectors, for example a=(t,x,y,z) where t,x,y,z are all Real coordinate coefficients, can be represented in Clifford notation as a=t+xe+yf+zg . [ ]

3D Rotation uses v={e,f,g}xImaginary, resulting in the transformation formula:

a'=exp(v) a exp(-v).

Notice that Clifford modulus of a defined as $|a|^2 = a a^\sim$ is an invariant of the Lorentz and rotation transforms [] :

$|a|^2 = t^2 - x^2 - y^2 - z^2$

> Note: Operator $( )^\sim$ is called Clifford Conjugation or Quaternion Conjugation defined as an involution that flips the signs of bases {e,f,g} and reverses the multiplication order. That is, for all a,b : $(ab)^\sim = b^\sim a^\sim$ and it toggles the signs of {e,f,g}. Example: $e^\sim=-e$, $1^\sim=1$, $i^\sim=i$, $i^\sim=(ef)^\sim = -fe = -i$.

Example of rotation

Let us use the following transform generator vector v:=i□□g :   a'=exp(i□□g) a exp(-i□□g), a=t+xe+yf+zg. We need to re-group the terms separating those that are commutative with g, and those that are anti-commutative with g: [ ]

a' = exp(i□□g)(t+zg)exp(-i□□g) + exp(i□□g)(xe+yf)exp(-i□□g) =

(t+zg) + exp(2i□□g)(xe+yf) =

(x cos2□□ + y sin2□□)e +

(-x sin2□□ + y cos2□□)f + t + zg

The first two terms at e and f are identical to a rotation matrix by angle 2□□ along the z (g) axis. The scalar t and z components are of course rotationally invariants, in this case.